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Function approximation of total system cost for a continuous manufacturing system

A case study

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Abstract *This paper presents a case study conducted in a sugar mill. The multi-level lot-sizing model suggested by Gunasekaran where the whole manufacturing system modelled as a function of total system cost (TSC) is used for this application. This study deals with minimising the TSC of a multi-stage, multi-facility and multi-product manufacturing system in which the buffer in between the stages is not allowed. The model is applied to find out the TSC and hence the fluctuating purchase prices of raw material. The consequence of implementing different costs that may occur due to change in government policies is analysed. This work has been carried out in an IBM/PC compatible system.*

Introduction

Multi-stage manufacturing inventory system is in essence the most common configuration in any manufacturing environment. Although the in-process inventory is inherent in any multi-stage manufacturing system, some systems do not allow the materials to be held in between stages. The control of in-process inventory, and especially its functional relationship to the manufacturing cycle time has also received a lot of attention in recent years. Taha and Skeith (1970) present one of the earliest lot-size models for multi-stage manufacturing which includes a process inventory. They permit any number of shipments between stages ignoring the cost of transporting batches. Jensen and Khan (1972) present a similar model without backlogging. Szendrovits (1975) optimised machine idle time under similar situations. Goyal and Gunasekaran (1990) studied the effect of dynamic process control on the economics of production. The same authors (Gunasekaran *et al.*, 1993) then suggested a direct pattern search method of finding out and implementing the economic lot sizing for a multi-stage, multi-facility and multi-product manufacturing inventory systems. But the problem frequently trapped within local minimal. Schwarz and Schrage (1975) have suggested a branch and bound algorithm for a multi-echelon manufacturing system. Hao *et al.* (1995) proposed a manufacturing controller consisting of two neural network structures.

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The mathematical model suggested by Gunasekaran *et al.* (1990) is modified so that the material will flow continuously through all facilities. Then the total system cost is approximated using feed forward neural network with back propagation algorithm. Normally, the sugar companies will fix the cost of raw material (sugarcane) in accordance with government policies. This will create misunderstanding between the farmers and the companies. The government also will purchase a part of sugar from the companies at a price fixed by the government. The company finds it difficult to fix the purchasing price of sugarcane either to the government or to the public market. The objective of this study is to analyse the past data regarding the seasonal variations in the price of sugarcane, the government purchase price and the processing cost. A feed forward neural network with back propagation algorithm is trained to see the pattern in these values, which in turn will give more accurate prices for the present conditions. In order to solve the problem in a real life Indian sugar company, the modified model has been used to investigate the level of success of the application.

The company

The company considered for the application of the model is one of the main manufacturers of sugar from sugarcane. This company is situated at the outskirts of Udumalpet, a small town in Tamilnadu. The capacity of the company is 1,250 tons of cane per day. A special officer administrates the factory. There are mainly four departments under him. There is an administration department, a canes department, an engineering department and a manufacturing department.

The administration department performs the establishment, account and purchase. The cane department looks after the cane cultivation by private farmers. The engineering department provides anchorage of processing cane juice to white sugar and by-products like press mud and final molasses.

Description of the manufacturing system

The harvested cane is crushed with the help of mills. The residual from the mills (badges) is used as fuel for the boilers. The juice (raw or mixed juice) is sent then to double sulphitation process. Sulphitation is done at two stages, first in the juice state and second in the syrup state. Raw juice from the mills is heated to 60°C to 65°C in the raw juice heaters. Then it is treated with milk lime ($Ca(OH)_2$) and sulphur dioxide. The resultant sulphured juice is heated to 100°C and sent to classifiers for setting. The juice overflowing from classifier is clear juice and free of suspended particles. The clear juice contains 85 per cent of water with pH value of 7 per cent to 8 per cent at 98°C. It is concentrated in evaporators to syrup. The syrup is once again sulphated and crystallised in pass. After crystallisation, sugar is dried, cooled and bagged. The above manufacturing process may be divided into six stages. The distinguishable stages are extraction, classification, evaporation, crystallisation, centrifugation and drying.

The desired production quantity is determined based on the on-hand inventory, orders on hand, capacity available and availability of raw materials. The mathematical model developed by Gunasekaran *et al.* (1990) has been identified as similar and the model was modified to suite for the application. The details of the model actually conferred for the study are presented hereunder.

Mathematical model

The mathematical model presented here deals with the lot-sizing problem of a multi-stage, multi-product production system. It is formulated by Gunasekaran *et al.* (1990). Here it is modified for multi-product and single stage system.

Assumptions

The following assumptions are made in the model:

- Demand for each product is known and deterministic.
- Capacities of the manufacturing facilities are fixed and known.
- Inventory holding cost is linear.
- Finished product inventory build-up is negligible.
- Product, stage, facility and feasibility combinations are deterministic and known.
- Operations time for product, stage and facility combinations are known.
- Transportation time of goods between any two stages is negligible.
- The value of unfinished product at the end of a stage is the value at the previous stage plus the value added at this stage.
- All the stages are combined to form one continuous system without any buffer in between them. This means that the in process inventory should not be allowed due to the various chemical reactions that take place continuously in manufacturing system.

Notations

i = Product index, $i = 1, 2 \ldots P$.

j = Stage index, $j = 1, 2 \ldots S$.

k = Machine/facility index, $k = 1, 2 \ldots n$.

BS_{ijk} = Batch size for item i at stage j and at machine k.

BSR_{ij} = Production run quantity for product i at stage j.

BL_{ijk} = Blocking quantity per spindle for product i at stage j for machine k.

C_{io} = Cost per unit of raw material for product i.

D_I = Demand rate for product i per unit time.

H = Inventory holding cost per unit investment (Rs) per unit time period which is independent of the products.

LF_{ijk} = Loading frequency for a batch of size BS_{ijk}.

n_j = Number of machines at stage j.

N_{ijk} = Number of spindles used per load for product i at stage j at machine k.

NC_{ij} = Number of production cycles for product i at stage j.

OP_{ijk} = Operation time per load (time unit) for product i at stage j for machine k.

P = Total number of products to be processed.

PQ_{ij} = Total production quantity for product i at stage j including the allowance for scrappage.

R_{ijk} = Set-up cost per spindle for product i at stage j and machine k.

S = Total number of production stages.

SC_{ijk} = Set-up cost per loading of product i.

SCP_{ij} = Set-up cost for the production run quantity BSR_{ij}.

SCR_{ij} = Scrape allowance for product i at stage k.

TSC = Total system cost.

WT_{ij} = Average waiting time for a production run quantity BSR_{ij}.

δ_{ijk} = Binary variable that takes the values 0 or 1. This variable indicates whether facility k at stage j can be used for product i.

Formulae

The aim of this model is to determine the optimal batch sizes at each machine by minimising the TSC. It consists of set-up cost,. inventory cost due to processing of batches and inventory cost due to waiting of batches.

Set-up cost. While loading the product on a machine each batch of size may be split in to a number of sub batches (of size $N_{ijk*}\mathrm{BL}_{ijk}$ units). The total set up cost for the given production quantity is the function of the loading frequency per batch and batch size. The loading for a batch depends on the number of spindles used and the batch blocking quantity for the spindle:

$$\text{Set-up cost, SC} = \sum_{i=1}^{P}\sum_{j=1}^{S} \mathrm{NC}_{ij}\mathrm{SCP}_{ij}$$

where,

$$\mathrm{SCP}_{ij} = \sum_{k=1}^{n} \mathrm{LF}_{ijk}\mathrm{SC}_{ijk}$$

$$\mathrm{LF}_{ijk} = \left|\frac{\mathrm{BS}_{ijk}}{\mathrm{BL}_{ijk}N_{ijk}}\right| + 0.9999$$

and

$$SC_{ijk} = N_{ijk}.R_{ijk}.$$

Inventory cost due to processing of batches. The in-process inventory carrying cost is incurred whenever a stock of half finished product is held. A product may be processed at more than one facility at a stage. For a product, different machines may be used at different times. The resultant production rate for a product at a stage is defined as the number of batches processed per unit time, considering all the facilities at that stage. This can be calculated by adding the individual production rates according to the feasibility of processing the product on various machines. Inventory cost due to processing of batches will be:

$$\sum_{i=1}^{P}\sum_{j=1}^{S} \mathrm{PQ}_{ij}\mathrm{C}_{\mathrm{io}}/2.H$$

and

$$\mathrm{PQ}_{ij} = D_i + \sum_{j=1}^{S} \mathrm{SCR}_{ij.}$$

Inventory cost due to waiting of batches at buffer storage. In this system, the waiting bay constitutes the buffer storage. The arrival process of batches of a product at the buffer storage depends on the processing rate at a stage j. The discharge process from the buffer depends upon the processing rate of the stage $j+1$. The total in-process inventory carrying cost due to the waiting of batches between stages is given by:

$$\text{Inventory cost due to waiting of batches ICWB} = \sum_{i=1}^{P}\sum_{j=1}^{S} \mathrm{PQ}_{ij}\mathrm{C}_{io}H.\mathrm{WT}_{ij}$$

$$\mathrm{WT}_{ij} = \mathrm{PQ}_{ij} + \frac{\sum_{k=1}^{n}\delta_{ijk}}{\mathrm{BSR}_{ij}}.$$

Then the TSC can be written as:

$$\sum_{i=1}^{P}\sum_{j=1}^{S}\mathrm{NC}_{ij}\mathrm{SCP}ij + \sum_{i=1}^{P}\sum_{j=1}^{S}\mathrm{PQ}_{ij}C_{io}/2.H + \sum_{i=1}^{P}\sum_{j=1}^{S}\mathrm{PQ}_{ij}C_{io}H.\mathrm{WT}_{ij}.$$

The proposed model deals with the approximating the TSC of continuous manufacturing system in which there is no buffer in between stages. In their mathematical model, Gunasekaran *et al.* (1990) considered continuous manufacturing system in which the materials are held up in between stages. If the nature of manufacturing restricts any stoppage in between the stages, the whole system can be modelled as single stage. Some modifications were made in the formulation and TSC may be simplified. The above derivation defines the TSC, which may consist of set-up and processing cost. So, for this application, the same may be assumed to consist of material, conversion and excess costs. The material cost is calculated from the purchasing of canes, purchase tax, cane less, transportation and cane development expenses. Conversion cost is the cost involved in purchasing the fuel, oil, lubricants and chemicals. Then excess cost may be derived from the by-product cost and miscellaneous income to the mill due to the processing. The summation of all the above costs will give the TSC per unit of time. The input for the calculation of various costs is collected from the past data available in the industry.

Solution methodology

The main objective is to approximate the TSC for the whole manufacturing process with the help of neural network architecture. The function approximation techniques were used. The outputs corresponding to some input vectors may be known from training data. But the mathematical function describing the actual process that generates the outputs from the input vector function approximation is the task of learning or constructing a function that generates approximately the same outputs from input vectors as the process being modelled based on available training data.

Figure 1 illustrates that the same finite set of samples can be used to obtain many different functions, all of which perform reasonably well on the given set of points. Since infinitely many functions exists that coincide for a finite set of points, additional criteria are necessary to decide which of these functions are

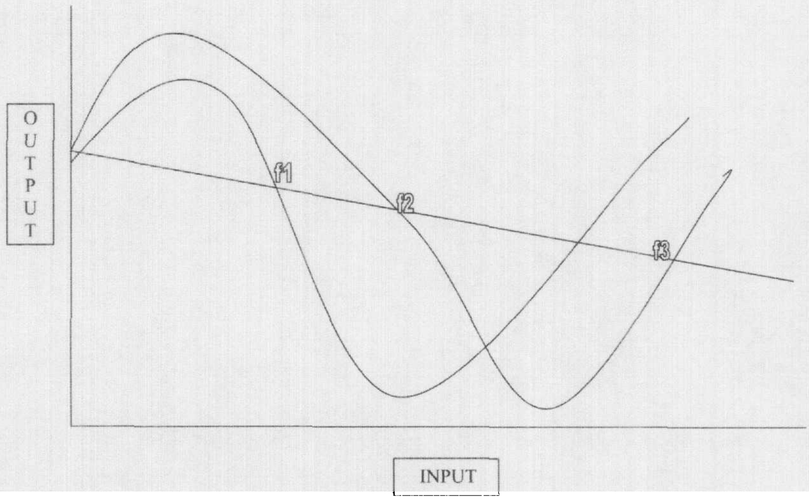

Figure 1.
Different functions obtained

desirable. Continuity and smoothness of the function are almost always required.

Function approximation can be performed using the neural networks. The back propagation training algorithm allows experimental acquisition of input/output mapping knowledge within multi-layer networks. The input patterns are submitted during the back propagation training sequentially. If a pattern is submitted and its classification or association is determined to be erroneous, the synaptic weights as well as the thresholds are adjusted so that the current least mean square classification error is reduced.

The input/output mapping comparison of target and actual values, the adjustment, if needed, continue until all mapping examples from the training set are learned within an acceptable overall error. Usually, mapping error is cumulative and computed over the full training set. During the association phase, the trained neural network itself operates in a feed forward manner. However the weight adjustments enforced by the learning rules propagate exactly backward from the output layer throw the hidden layer towards the input layer.

The problem was solved using the software MATLAB version 5.1. Neural network is one of the toolbox available in the MATLAB. A program was written using MATLAB functions for approximating the function.

Results and analysis

The neural network will reach its error goal quickly if the mapping functions are having values close to 1. So the actual date collected from the industry were modified. The results shown in Table I are the comparison of actual outputs i.e. target with the trained network output for maximum epoch = 25,000, err-goal = 0.0001 and the learning rate = 0.2.

Cane purchase cost (Rs)	Total manufacturing cost (× 1000Rs)	Total manufacturing cost given by network (× 1,000Rs)
700	1.6095	1.6044
711.20	1.6207	1.6170
722.40	1.6320	1.6295
733.60	1.6431	1.6419
744.80	1.6543	1.6532
756.0	1.6655	1.6662
767.20	1.6777	1.6782
778.40	1.6880	1.6900
789.60	1.6991	1.7017
800.80	1.7103	1.7132
812.00	1.7220	1.7246
823.20	1.7327	1.7358
834.40	1.7440	1.7469
845.60	1.7531	1.7579
856.80	1.7663	1.7687
868.00	1.7775	1.7794
879.20	1.7890	1.7900
890.40	1.7999	1.8004
901.60	1.8111	1.8107
912.80	1.8223	1.8209
924.00	1.8340	1.8309
935.20	1.8447	1.8409
946.40	1.8559	1.8506

Table I. Comparison of actual outputs

Time periods for one year	First four months	Second four months	Third four months	Last four months
Existing price in Rs	2,58,974	2,14,698	2,58,637	2,86,424
Purchasing price from neural network in Rs	2,28,725	1,85,778	2,24,884	2,49,589
Percentage savings	11.68	13.47	13.05	12.86

Table II. The savings for the company during the one year

Variable FOL. cost	Material cost in	Conversion cost	Overhead cost	Excess cost	Total system cost
0.01717	0.818	0.0911	0.8097	0.0334	1.6928
0.034	0.818	0.1167	0.8097	0.0334	1.7104
0.059	0.818	0.1337	0.8097	0.0334	1.7274

Note: All the costs are given in Rupees

Table III. Variation in the total system cost for various costs of fuel, oil and lubricants

At one time, the cane purchase cost during the year 1997-1998 in the mill was Rs733-60 per ton. For this cost, the total manufacturing cost given by the neural network is Rs. 1,653-20 per ton. In the mill they are crushing 1,250 tons per day. So the total manufacturing cost can be approximately reduced by Rs1,500 per day. Hence the company can fix the new purchasing price of the cane, which is found to be more suitable to both farmers and the company. This enables the company in better position. Table II shows the savings for the company during the past year.

Now this trained network can be used to approximate the total system cost for any variation in its constituents. The trained neural network responds to the changes in any one of the variables of the total system cost. For example the material cost during the year 1997-1998 was kept constant (Rs818.00). Table III shows the variation in the total system cost for various costs of fuel, oil and lubricants.

Conclusion

For the training neural network problem, past data from the industry were used. Since the mathematical model gives approximate costs, the network approximation still refines the final results. With the constraints regarding the availability of the data required and the operational features understood of such a specialised system, the attempt to apply the model could be considered successful. The results obtained are operationally feasible however, only with certain careful considerations and perhaps suitable modifications. The total cost corresponding to the results obtained has been compared with a rough estimate of the costs incurred with present practice, which indicates less in the value of TSC. This analysis of these results also brought to the fore the limitations of applying to such a large system without any regards to various operational constraints. The experience gained certainly indicates that there is potential to apply the model.

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